Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Evaluate $\displaystyle\int_e^{e^2} \frac{4}{x \ln x}\, dx.$

Solution: This is a u-substitution problem. Let $u = \ln x$, $du = \dfrac{1}{x}\, dx \Rightarrow$ $x\, du = dx$. We do have bounds, which means we need to convert:
$$u(e) = \ln e = 1 \quad \text{and} \quad u(e^2) = \ln e^2 = 2.$$
Thus, we write
$$\int_e^{e^2} \frac{4}{x \ln x}\, dx = \int_1^2 \frac{4}{xu}(x\, du)$$
$$= \int_1^2 \frac{4}{u}\, du$$
$$= 4 \ln u \Big|_1^2$$
$$= 4 \ln 2 - 4 \underbrace{\ln 1}_{0}$$
$$= \boxed{4 \ln 2}$$

2. [5 pts] Evaluate $\displaystyle\int_e^{e^2} 4x \ln x\, dx.$

Solution: This is an integration by parts problem. By LIATE, let $u = \ln x$ and so $dv = 4x\, dx$. Our table is
$$
\begin{array}{ll}
u = \ln x & dv = 4x\, dx \\
du = \dfrac{1}{x}\, dx & v = 2x^2
\end{array}.
$$
Our integral becomes

$$\int_e^{e^2} 4x \ln x \, dx = \underbrace{\ln x}_{u} \underbrace{(2x^2)}_{v} \Big|_e^{e^2} - \int_e^{e^2} \underbrace{2x^2}_{v} \underbrace{\left(\frac{1}{x} dx\right)}_{du}$$

$$= 2x^2 \ln x \Big|_e^{e^2} - \int_e^{e^2} 2x \, dx$$

$$= 2x^2 \ln x \Big|_e^{e^2} - x^2 \Big|_e^{e^2}$$

$$= 2x^2 \ln x - x^2 \Big|_e^{e^2}$$

$$= 2(e^2)^2 \underbrace{\ln e^2}_{2} - (e^2)^2 - [2(e)^2 \underbrace{\ln e}_{1} - (e)^2]$$

$$= 4e^4 - e^4 - [2e^2 - e^2]$$

$$= \boxed{3e^4 - e^2}$$